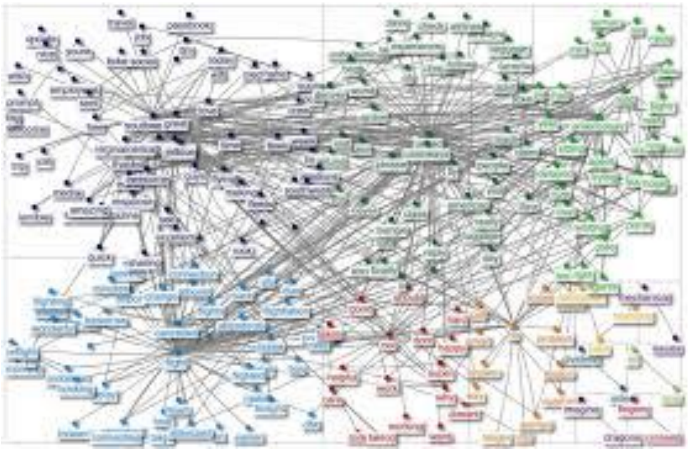
An Idea Calculus for the Constructive Approach

Ioachim Drugus
Institute of Mathematics and Computer Science
Chișinău, Moldova
ioachim.drugus@math.md

Three Periods in Development of Artificial Intelligence

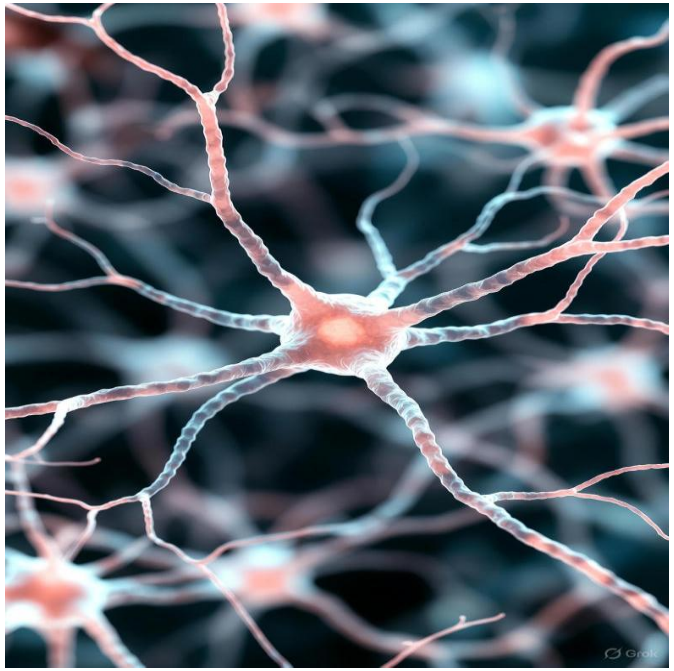
Classical
Based on Logic

Expert Systems
Knowlege Bases
Semantic Networks
Semantic Web
Ontologies



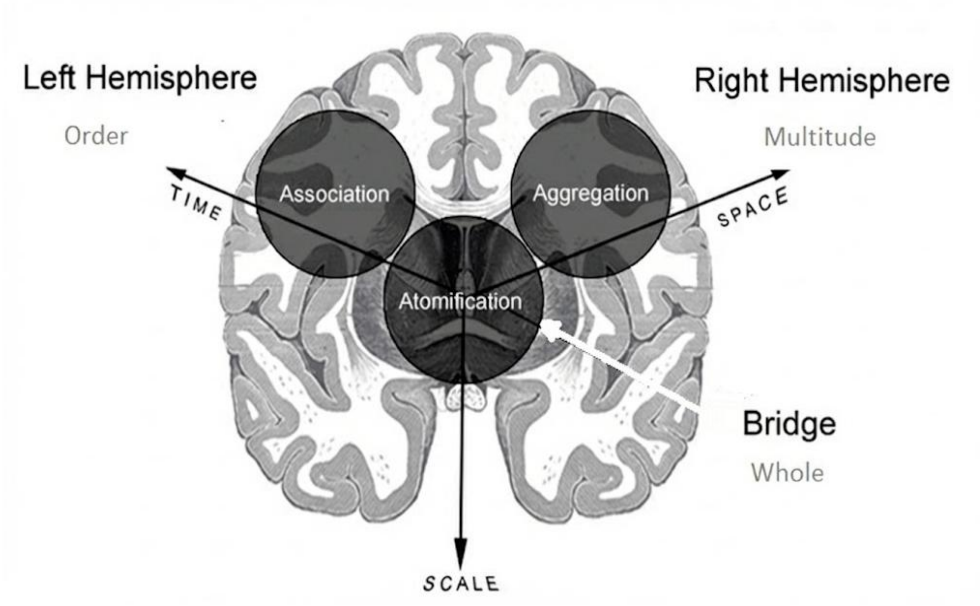
Current
Based on Machine Learning

A Logical Calculus of the Ideas
Immanent in Nervous Activity
by Warren McCulloch and Walter Pitts
(1943)



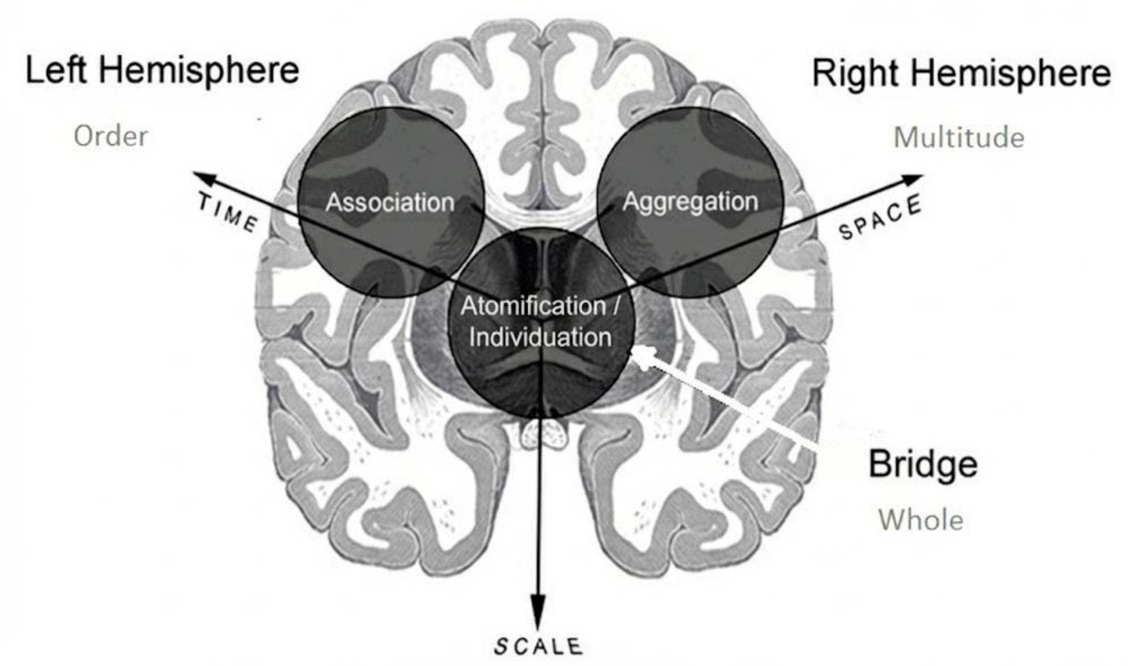
Future
Based on Logic and AI

A Whole Brain approach to the Web
by Ioachim Drugus
(2007)



An Idea Calculus for Natural and Artificial Intelligence
(2025)

Towards a Logical Calculus of Ideas Immanent in Whole Brain Activity



(1) ***Association***, a binary operation which, given two ideas ***s*** and ***t***, produces an association link from ***s*** to ***t***, which together with ***s*** and ***t***, make up an *association pair* denoted as (***s***, ***t***).

(2) ***Aggregation***, a binary operation, which given two ideas ***s*** and ***t***, produces their aggregation.

(3) ***Atomification (individuation)***, a unary operation, which for an idea **s** produces a mereological atom.

Association can be precisely modeled by the set theoretic ordered pair
Aggregation produces only the content of the set {s,t}
Individuation of an idea ***s*** will be denoted as ***s***° (grade)

What is an idea?

An idea **i** has an ***extension*** ***Ext*** (***i***)– the class of things presented by idea *i* and an *intension* ***Int***(***i***) whereby the idea ***i*** is distigiuished from other ideas

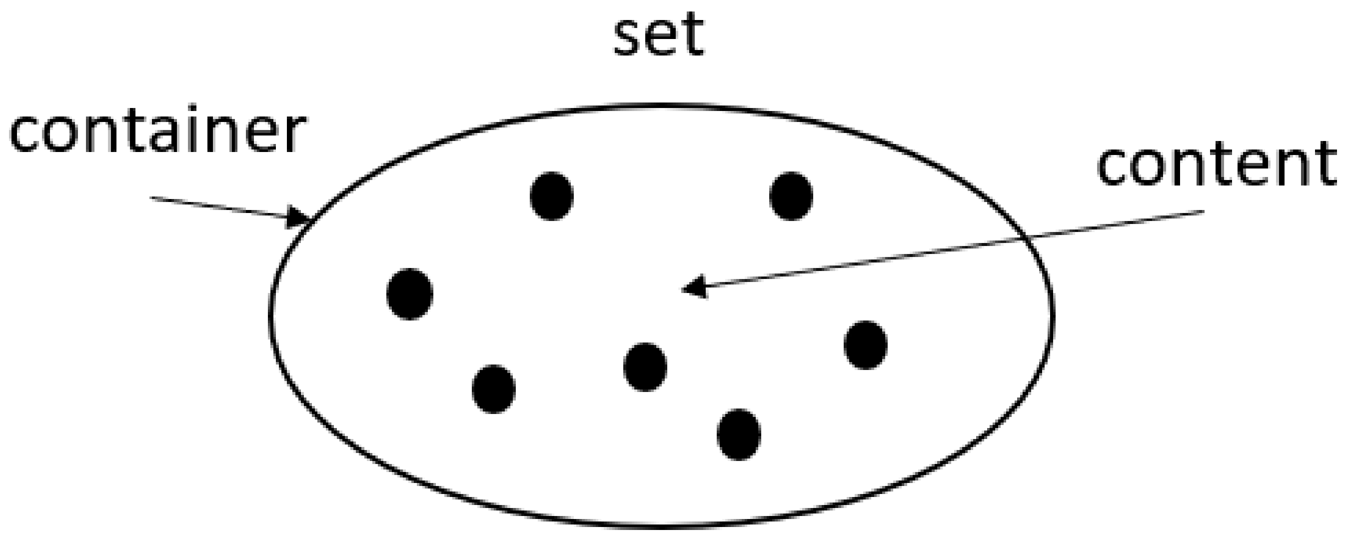
The *idea of* chair *represents* members of idea's extension recognizable by a *pattern* (without understanding)

as well as members of this class recognizable by compliance to a *description* (with understanding)

John Locke: A*n Essay Concerning Humane Understanding* Thomas Basset. London (1690):
Ideas are **objects** of understanding when a man thinks.

As per John Locke, current days AI based only on machine learning cannot have ideas, it does not think. Only with logic AI could understand, only with universal logic AI would fully understand.

Mereological view on sets and classes



A proper class has no container

- A set theory is *built upon mereology* if the subset symbol „$\subseteq$” (rather than membership symbol „$\in$”) is used as primitive. Only the primitive „$\subseteq$” would not suffice for a set theory built upon mereology, also the individuation operation symbol must be taken as primitive.
- However, in a set theory built upon mereology *with atoms* the subset symbol „$\subseteq$” cannot be taken as primitive because a notation of atom on any side of this symbol would create confusion -- another symbol should be used
- In this presentation the partial order symbol „$\leq$”is used for parthood.

On extensions

Idea Calculus (IC) = Extension Theory Minus Extensionality

Extension Theory is a set theory in algebraic presentation with the ***Extensionality*** axiom:

$$\forall z\ (z \in x \leftrightarrow x \in y) \rightarrow x = y$$

- ***IC*** is proposed as a logical foundation for intelligence science
- ***IC*** offers an object-oriented approach to intelligence; **IC** treats ***ideas*** as objects of mind
- ***IC*** is the weakest intensional set theory

Extension Theory

Drugus, I. *A Universal Algebraic Set Theory Built on Mereology with Applications*. Logica Universalis. 16, pp. 253–283. Springer Nature (2022).

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$x \leq y \fallingdotseq x \cup y = y;$ (partial order relation); (8.1)

$x \in y \fallingdotseq x^\circ \leq y$ (membership relation); (8.2)

$\{x\} \fallingdotseq x^\circ$ (synonymic notation); (8.3)

$x \triangleright y \fallingdotseq x \cup \{y\}$ (adjunction operation); (8.4)

$x \bigcirc y \fallingdotseq \{x\} \triangleright y;$ (aggregation operation). (8.5)

The list of ET's axioms consists of three groups of axioms:

1. *Ground universe* group of axioms (obtained by converting into the symbols of ET the axioms of AGG):

$$a \setminus a = \varnothing, \quad (8.6)$$
$$a \cup (b \cup c) = (a \cup b) \cup c, \quad (8.7)$$
$$a \cup b = b \cup a, \quad (8.8)$$
$$a \cup a = a, \quad (8.9)$$
$$(a \cup b) \setminus c = (a \setminus c) \cup (b \setminus c), \quad (8.10)$$
$$a \setminus (b \cup c) = (a \setminus b) \setminus c, \quad (8.11)$$
$$a \cup (b \setminus a) = a \cup b, \quad (8.12)$$
$$a \cup (a \setminus b) = a, \quad (8.13)$$
$$(a \setminus b) \setminus c = (a \setminus c) \setminus (b \setminus c), \quad (8.14)$$
$$a \setminus (b \setminus c) = (a \setminus b) \cup (a \setminus (a \setminus c)). \quad (8.15)$$

2. *Structure universe* group of axioms:

$$x^\circ = y^\circ \rightarrow x = y \quad (8.16)$$
$$\forall z (z \in x \leftrightarrow z \in y) \rightarrow x = y \quad (8.17)$$

3. *Union* group of axioms:

$$y \in x \rightarrow y \leq \bigcup x \quad (8.18)$$
$$(\forall y \in x)(y \leq z) \rightarrow \bigcup x \leq z \quad (8.19)$$

Idea Calculus (Right Hemisphere – Extension Theory)

1. *Algebraic features*:

$$a \setminus a = \emptyset, \tag{3.4}$$

$$a \cup (b \cup c) = (a \cup b) \cup c, \tag{3.5}$$

$$a \cup b = b \cup a, \tag{3.6}$$

$$a \cup a = a, \tag{3.7}$$

$$(a \cup b) \setminus c = (a \setminus c) \cup (b \setminus c), \tag{3.8}$$

$$a \setminus (b \cup c) = (a \setminus b) \setminus c, \tag{3.9}$$

$$a \cup (b \setminus a) = a \cup b, \tag{3.10}$$

$$a \cup (a \setminus b) = a, \tag{3.11}$$

$$(a \setminus b) \setminus c = (a \setminus c) \setminus (b \setminus c), \tag{3.12}$$

$$a \setminus (b \setminus c) = (a \setminus b) \cup (a \setminus (a \setminus c)). \tag{3.13}$$

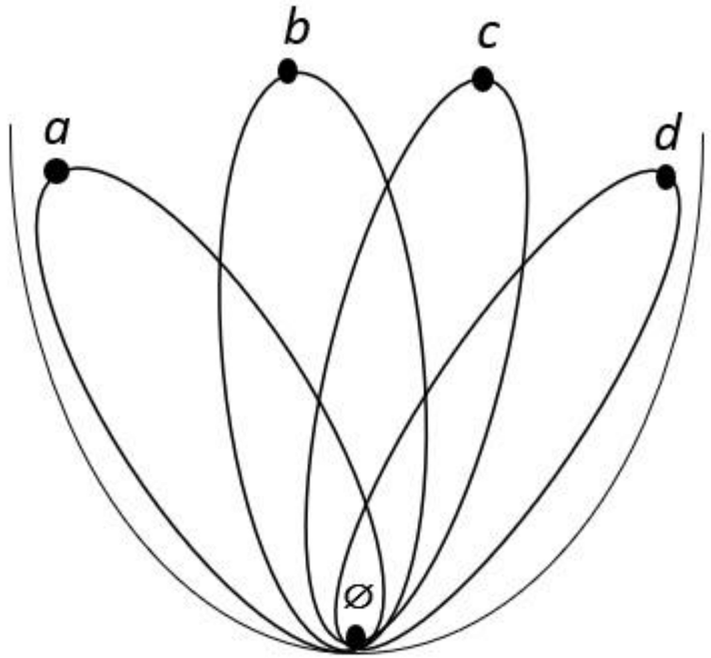
2. *Individuation feature*:

$$x^\circ = y^\circ \to x = y \tag{3.14}$$

3. *Generalization features*:

$$y \in x \to y \subseteq \bigcup x \tag{3.15}$$

$$(\forall y \in x)(y \subseteq z) \to \bigcup x \subseteq z \tag{3.16}$$



Generalized Boolean Algebra

Idea Calculus (Left Hemisphere – Imbrication Theory)

Definition 1. (see [7]) The *imbrication theory* is a theory of universal algebras, named *imbrication algebras*, in the following signature of three operations:

- binary *pairing operation*, with empty prefix symbol;
- unary *left projection*, with superscript symbol "+";
- unary *right projection*, with superscript symbol "×",

with the following three axioms:

$$(x,\ y) = (x',\ y') \to x = x' \ \&\ y = y' \ (\textit{pairing axiom});$$

$$(x, y)^+ = x; \tag{4.1}$$

$$(x, y)^\times = y\,. \tag{4.2}$$

The pairing axiom is equivalent to the conjunction of the universal closures of the following two formulas:

$$(x, y) = (x', y) \to x = x',$$
$$(x, y) = (x, y') \to y = y'.$$

Theorem. The class of imbrication algebras is a quasi-variety

Hence this class is closed under isomorphic images, subulgebras, direct products, ultraproducts

Main example of imbrication algebras are Johansson-Tarski algebras

My Cycle of Works on Ideas

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2. Drugus, I., *Metalingua – a Formal Language for Integration of Disciplines via their Universes of Discourse*. Economy Transdisciplinarity Cognition Journal, Vol. 12, N2 (2009), pp. 17–23
3. Drugus, I., : *Universics – a Structural Framework for Knowledge Representation*. In: Knowledge Engineering Principles and Techniques. Cluj-Napoca, Romania, pp. 115–118 (2009)
4. Drugus, I., *Universics: an Approach to Knowledge based on Set theory*. In: Knowledge Engineering Principles and Techniques. Selected Extended Papers. Cluj-Napoca, Romania, pp. 193–200 (2009)
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7. Drugus, I., : *Metalingua*: A Language to Mediate Communication with Semantic Web in Natural Languages. K.S. Thaung (Ed.): Advanced Information Technology in Education, AISC 126, 2011.Springer-Verlag Berlin Heidelberg (2012), pp. 109–115.
8. Drugus I, *PML: A Punctuation Symbolism for Semantic Markup*. Proc. of 11th International Conf. “Linguistic Resources and Tools for Processing the Romanian Language”, 26-27 November 2015, pp. 79 – 92.
9. Drugus, I., *Towards an ontology of individuals: Comments on “Identity, ontology and Frege’s problem” of William Greenberg*, Comput. Sci. J. Moldova 23 (2015), no. 1, 92–96; MR3353524.
10. Drugus, I., *Universics: an Axiomatic Theory of Universes for the Foundations (in two parts)*. In: Proceedings of the Workshop on Foundations of Informatics, August 24-29, 2015, Chisinau, Republic of Moldova. pp. 118-153.
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12. Drugus I. *Towards an Algebraic Set Theory for Formalization of Data Structures*. Proc. MFOI2018, July 2-6, 2018, Chisinau, Moldova. pp. 73-89.
13. Drugus, I., Skobelev, V. G. *Imbrication algebras – algebraic structures of nesting order*. CSJM, vol.26, no.3(78), (2018) pp. 233-250.
14. Drugus, I., Skobelev, V. G. *Some developments of Aggregate Theory*. Proceedings of the Conference on Mathematical Foundations of Informatics MFOI2019, July 3-6, 2019, Iasi, Romania.
15. Drugus, I. *Towards a Non-associative Model of Language*. Proceedings of the Conference on Mathematical Foundations of Informatics MFOI2020, January 12-16, 2021, Kyiv, pp 123-134.
16. Drugus, I. A Universal Algebraic Set Theory Built on Mereology with Applications. Logica Universalis 16, 253–283, 2022
17. Drugus, I. Idea Theory: Towards Logical Foundations of Intelligence Science. Proceedings of the Future Technologies Conference (FTC), 344–356. 2023.
18. Drugus I, An Idea Calculus for Natural and Artificial Intelligence, Logica Universalis, 2025.

Thank you!